تمرين رقم 90 صفحة 34 الكتاب المدرسي الجزء الأول

بين باستعمال طريقة مناسبة أن :

- $$\lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sqrt{1-\cos x}} = 2\sqrt{2}$$
- $$\lim_{x \to \frac{\pi}{2}} (\pi - 2x)\tan x = 2$$
- $$\lim_{x \to 0} \frac{\sin x}{\sqrt{x+1}-1} = 2$$

الحل

أولا $$\lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sqrt{1-\cos x}} = 2\sqrt{2}$$

لدينا $\lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sqrt{1-\cos x}} = \frac{0}{0}$ وهي احدى حالات عدم التعين $\frac{0}{0}$

إزالة حالة عدم التعين

$$\lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sqrt{1-\cos x}} = \lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sqrt{1-\cos x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}$$

$$\lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sqrt{1-\cos x}} = \lim_{x \xrightarrow{>} 0} \frac{\sin 2x\sqrt{1+\cos x}}{\sqrt{(1-\cos x)(1+\cos x)}}$$

$$\lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sqrt{1-\cos x}} = \lim_{x \xrightarrow{>} 0} \frac{\sin 2x\sqrt{1+\cos x}}{\sqrt{1-\cos^2 x}}$$

$$\lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sqrt{1-\cos x}} = \lim_{x \xrightarrow{>} 0} \frac{\sin 2x\sqrt{1+\cos x}}{\sqrt{\sin^2 x}}$$

نذكر أن $1-\cos^2 x = \sin^2 x$

$$\lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sqrt{1-\cos x}} = \lim_{x \xrightarrow{>} 0} \frac{\sin 2x\sqrt{1+\cos x}}{\sin x}$$

$$\lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sqrt{1-\cos x}} = \lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sin x}\sqrt{1+\cos x}$$

نذكر أن $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$

$$\lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sqrt{1-\cos x}} = 2\sqrt{1+\cos 0} = 2\sqrt{2}$$

ومنه $$\lim_{x \xrightarrow{>} 0} \frac{\sin 2x}{\sqrt{1-\cos x}} = 2\sqrt{2}$$

ثانيا $\lim_{x \to \frac{\pi}{2}} (\pi - 2x)\tan x = 2$

لدينا $\lim_{x \to \frac{\pi}{2}} (\pi - 2x)\tan x = 0 \times (+\infty)$ وهي احدى حالات عدم التعين $0\times(+\infty)$

ازالة حالة عدم التعين

$$\lim_{x \to \frac{\pi}{2}} (\pi - 2x)\tan x = \lim_{x \to \frac{\pi}{2}} (\pi - 2x)\frac{\sin x}{\cos x}$$

$$\lim_{x \to \frac{\pi}{2}} (\pi - 2x)\tan x = \lim_{x \to \frac{\pi}{2}} \frac{\sin x}{\frac{\cos x}{(\pi - 2x)}}$$

$$\lim_{x \to \frac{\pi}{2}} (\pi - 2x)\tan x = \lim_{x \to \frac{\pi}{2}} \frac{\sin x}{\frac{\cos x}{-2\left(x - \frac{\pi}{2}\right)}}$$

$$\lim_{x \to \frac{\pi}{2}} (\pi - 2x)\tan x = \lim_{x \to \frac{\pi}{2}} -2 \times \frac{\sin x}{\frac{\cos x}{\left(x - \frac{\pi}{2}\right)}}$$

نذكر ان $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\left(x - \frac{\pi}{2}\right)} = -\sin\frac{\pi}{2} = -1$

$$\lim_{x \to \frac{\pi}{2}} (\pi - 2x)\tan x = -2 \times \frac{\sin\frac{\pi}{2}}{-\sin\frac{\pi}{2}}$$

$$\lim_{x \to \frac{\pi}{2}} (\pi - 2x)\tan x = -2 \times -1 = 2$$

ومنه $\lim_{x \to \frac{\pi}{2}} (\pi - 2x)\tan x = 2$

ثالثا $\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-1}=2$

لدينا $\frac{0}{0}=\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-1}$ وهى احدى حالات عدم التعين $\frac{0}{0}$

إزالة حالة عدم التعين

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-1}=\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-1}\times\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$$

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-1}=\lim_{x\to 0}\frac{\sin x\sqrt{x+1}+1}{\left(\sqrt{x+1}\right)^2-1}$$

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-1}=\lim_{x\to 0}\frac{\sin x\left(\sqrt{x+1}+1\right)}{x+1-1}$$

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-1}=\lim_{x\to 0}\frac{\sin x}{x}\left(\sqrt{x+1}+1\right)$$

نذكر ان $\lim_{x\to 0}\frac{\sin x}{x}=1$ ومنه

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-1}=1\times\left(\sqrt{1}+1\right)=2$$

ومنه ثالثا $\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-1}=2$

تم بحمد الله و فضله