

تمرين رقم 89 صفحة 34 الكتاب المدرسي الجزء الأول

باستعمال تعريف العدد المشتق عند $\frac{\pi}{4}$ لكل من الدالتين $x \mapsto 2 \cos x - \sqrt{2}$ و $x \mapsto \tan x$ ،

احسب النهاية التالية $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{2 \cos x - \sqrt{2}}$

الحل

نضع $f(x) = 2 \cos x - \sqrt{2}$ ومنه $f\left(\frac{\pi}{4}\right) = 2 \cos \frac{\pi}{4} - \sqrt{2} = 2 \frac{\sqrt{2}}{2} - \sqrt{2} = 0$

ومنه $f'(x) = -2 \sin x$ إذن $f'\left(\frac{\pi}{4}\right) = -2 \sin \frac{\pi}{4} = -2 \frac{\sqrt{2}}{2} = -\sqrt{2}$

إذن حسب تعريف العدد المشتق للدالة f عند الصفر $\frac{\pi}{4}$ نلاحظ

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{f(x) - f\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 \cos x - \sqrt{2}}{x - \frac{\pi}{4}} = f'\left(\frac{\pi}{4}\right)$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{f(x) - f\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}} = -2 \sin \frac{\pi}{4}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{f(x) - f\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}} = -2 \frac{\sqrt{2}}{2}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{f(x) - f\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}} = -\sqrt{2}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{2 \cos x - \sqrt{2}}{x - \frac{\pi}{4}} = -\sqrt{2} \quad \text{ومنه}$$

ثانيا

$$g\left(\frac{\pi}{4}\right) = \tan \frac{\pi}{4} = 1 \quad \text{نضع } g(x) = \tan x \quad \text{ومنه}$$

$$g'\left(\frac{\pi}{4}\right) = \frac{1}{\cos^2\left(\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = 2 \quad \text{ومنه نضع } g'(x) = \frac{1}{\cos^2 x} \quad \text{ومنه}$$

إذن حسب تعريف العدد المشتق للدالة g عند الصفر $\frac{\pi}{4}$ نلاحظ

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{g(x) - g\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{g(x) - g\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 \cos x - 1}{x - \frac{\pi}{4}} = g'\left(\frac{\pi}{4}\right)$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{g(x) - g\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}} = 2$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}} = 2 \quad \text{إذن}$$

لدينا $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{2 \cos x - \sqrt{2}}$ و هي من حالات عدم التعيين من الشكل $\frac{0}{0}$.

إزالة حالة عدم التعيين

نقسم البسط والمقام على $\left(x - \frac{\pi}{4}\right)$ ومنه نجد

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{2 \cos x - \sqrt{2}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\frac{\tan x - 1}{x - \frac{\pi}{4}}}{\frac{2 \cos x - \sqrt{2}}{x - \frac{\pi}{4}}}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin 3x}{2 \cos x - 1} = \frac{g' \left(\frac{\pi}{4} \right)}{f' \left(\frac{\pi}{4} \right)}$$

لدينا

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin 3x}{2 \cos x - 1} = \frac{2}{-\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{2 \cos x - \sqrt{2}} = -\frac{\sqrt{2}}{2} \quad \text{إذن}$$

تم بحمد الله