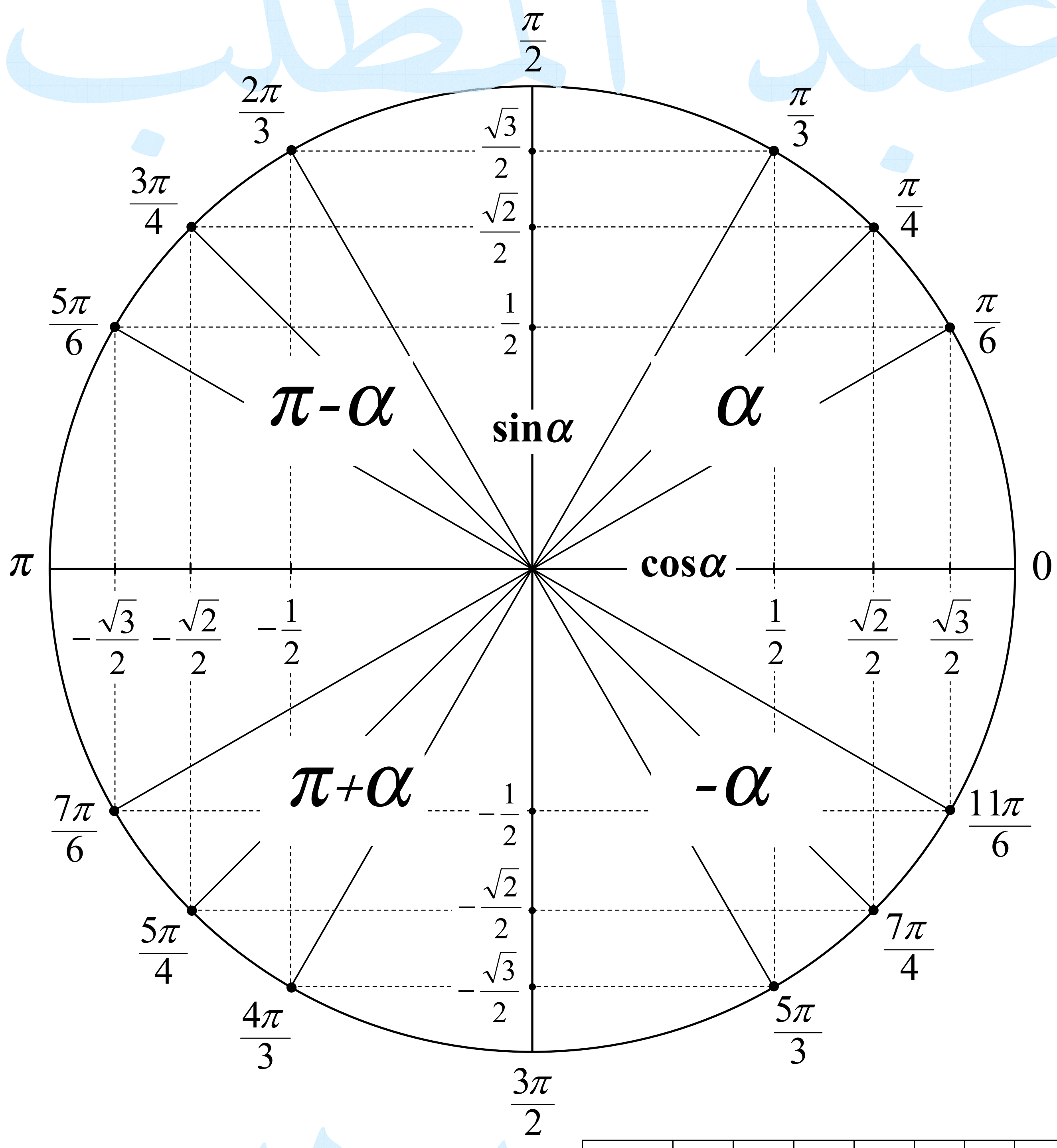
إعداد: عبد المطلب

بوزريعة: محرم 1432هـ

Cercle Trigonométrique



$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1$$
$$= 1 - 2\sin^2\alpha$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$2\cos^2\alpha = 1 + \cos 2\alpha$$

$$2\sin^2\alpha = 1 - \cos 2\alpha$$

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$tg\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞

Fonctions Trigonométriques et hyperboliques

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
sinα	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
cosα	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
tgα	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞

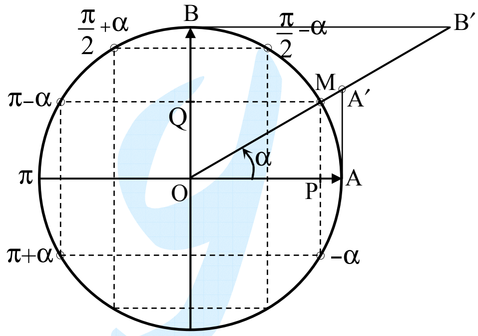
$\cos\alpha = \overline{OP}$ $\quad -1 \le \cos\alpha \le 1$

$\sin\alpha = \overline{OQ}$ $\quad -1 \le \sin\alpha \le 1$

$$\text{tg}\alpha = \overline{AA'} = \frac{\sin\alpha}{\cos\alpha} = \frac{1}{\text{cotg}\alpha}$$

$$\text{cotg}\alpha = \overline{BB'} = \frac{\cos\alpha}{\sin\alpha} = \frac{1}{\text{tg}\alpha}$$

$$\sec\alpha = \frac{1}{\cos\alpha}\ ;\ \text{cosec}\alpha = \frac{1}{\sin\alpha}$$



$$\sin^2\alpha + \cos^2\alpha = 1$$

$$1 + \text{tg}^2\alpha = \frac{1}{\cos^2\alpha}$$

$$1 + \text{cotg}^2\alpha = \frac{1}{\sin^2\alpha}$$

$\cos(\alpha + 2k\pi) = \cos\alpha$

$\sin(\alpha + 2k\pi) = \sin\alpha$

$\text{tg}(\alpha + k\pi) = \text{tg}\alpha$

$\cos(-\alpha) = \cos\alpha$

$\sin(-\alpha) = -\sin\alpha$

$\text{tg}(-\alpha) = -\text{tg}\alpha$

• k nombre relatif •

$\cos(\pi + \alpha) = -\cos\alpha$

$\sin(\pi + \alpha) = -\sin\alpha$

$\text{tg}(\pi + \alpha) = \text{tg}\alpha$

$\cos(\pi - \alpha) = -\cos\alpha$

$\sin(\pi - \alpha) = \sin\alpha$

$\text{tg}(\pi - \alpha) = -\text{tg}\alpha$

$\cos(\frac{\pi}{2} - \alpha) = \sin\alpha$

$\sin(\frac{\pi}{2} - \alpha) = \cos\alpha$

$\text{tg}(\frac{\pi}{2} - \alpha) = \frac{1}{\text{tg}\alpha}$

$\cos(\frac{\pi}{2} + \alpha) = -\sin\alpha$

$\sin(\frac{\pi}{2} + \alpha) = \cos\alpha$

$\text{tg}(\frac{\pi}{2} + \alpha) = -\frac{1}{\text{tg}\alpha}$

$\sin\alpha = 0 \Leftrightarrow \alpha = k\pi$

$\cos\alpha = 0 \Leftrightarrow \alpha = \frac{\pi}{2} + k\pi$

$\sin\alpha = 1 \Leftrightarrow \alpha = \frac{\pi}{2} + 2k\pi$

$\cos\alpha = 1 \Leftrightarrow \alpha = 2k\pi$

$\sin\alpha = -1 \Leftrightarrow \alpha = \frac{3\pi}{2} + 2k\pi$

$\cos\alpha = -1 \Leftrightarrow \alpha = \pi + 2k\pi$

• k nombre relatif •

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1$

$= 1 - 2\sin^2\alpha = \frac{1 - \text{tg}^2\alpha}{1 + \text{tg}^2\alpha}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\,\text{tg}\alpha}{1 + \text{tg}^2\alpha}$

$2\cos^2\alpha = 1 + \cos 2\alpha$

$2\sin^2\alpha = 1 - \cos 2\alpha$

$4\cos^3\alpha = \cos 3\alpha + 3\cos\alpha$

$4\sin^3\alpha = -\sin 3\alpha + 3\sin\alpha$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\text{tg}(\alpha+\beta) = \frac{\text{tg}\alpha + \text{tg}\beta}{1 - \text{tg}\alpha\,\text{tg}\beta}$

$\text{tg}(\alpha-\beta) = \frac{\text{tg}\alpha - \text{tg}\beta}{1 + \text{tg}\alpha\,\text{tg}\beta}$

$\text{tg}\,2\alpha = \frac{2\,\text{tg}\alpha}{1 - \text{tg}^2\alpha}$

$\cos\alpha = \cos\beta \Leftrightarrow \begin{cases} \alpha = \beta + 2k\pi \\ \quad V \\ \alpha = -\beta + 2k\pi \end{cases}$

$\sin\alpha = \sin\beta \Leftrightarrow \begin{cases} \alpha = \beta + 2k\pi \\ \quad V \\ \alpha = \pi - \beta + 2k\pi \end{cases}$

$\text{tg}\alpha = \text{tg}\beta \Leftrightarrow \alpha = \beta + k\pi$

• k nombre relatif •

$\cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$

$\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}$

$e^{i\alpha} = \cos\alpha + i\sin\alpha$

$e^{-i\alpha} = \cos\alpha - i\sin\alpha$

e =2,718281828...

i nbre imag: $i^2 = -1$

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\text{tg}\alpha\,\text{tg}\beta = \frac{\text{tg}\alpha + \text{tg}\beta}{\text{cotg}\alpha + \text{cotg}\beta} = \frac{\text{tg}\beta - \text{tg}\alpha}{\text{cotg}\alpha - \text{cotg}\beta}$

$a\cos\alpha + b\sin\alpha = \sqrt{a^2+b^2}\,\sin(\alpha+\theta)$

$\sin\theta = \frac{a}{\sqrt{a^2+b^2}}\ ;\ \cos\theta = \frac{b}{\sqrt{a^2+b^2}}$

$a\cos\alpha + b\sin\alpha = \sqrt{a^2+b^2}\,\cos(\alpha-\theta)$

$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}\ ;\ \sin\theta = \frac{b}{\sqrt{a^2+b^2}}$

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$\text{tg}\alpha + \text{tg}\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$

$\text{tg}\alpha - \text{tg}\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$

$\text{ch}\alpha = \frac{e^{\alpha} + e^{-\alpha}}{2}\ ;\ \text{sh}\alpha = \frac{e^{\alpha} - e^{-\alpha}}{2}$

$\text{th}\alpha = \frac{\text{sh}\alpha}{\text{ch}\alpha} = \frac{e^{\alpha} - e^{-\alpha}}{e^{\alpha} + e^{-\alpha}}$

$\text{cth}\alpha = \frac{\text{ch}\alpha}{\text{sh}\alpha} = \frac{e^{\alpha} + e^{-\alpha}}{e^{\alpha} - e^{-\alpha}}$

$\text{ch}\alpha + \text{sh}\alpha = e^{\alpha}\ ;\ \text{ch}\alpha - \text{sh}\alpha = e^{-\alpha}$

$\text{ch}^2\alpha - \text{sh}^2\alpha = 1\ ;\ \text{th}\alpha\,\text{cth}\alpha = 1$

$\text{sh}(-\alpha) = -\text{sh}\alpha\ ;\ \text{ch}(-\alpha) = \text{ch}\alpha$

$\text{th}(-\alpha) = -\text{th}\alpha\ ;\ \text{cth}(-\alpha) = -\text{cth}\alpha$

$\text{sh}(\alpha\pm\beta) = \text{sh}\alpha\,\text{ch}\beta \pm \text{ch}\alpha\,\text{sh}\beta$

$\text{ch}(\alpha\pm\beta) = \text{ch}\alpha\,\text{ch}\beta \pm \text{sh}\alpha\,\text{sh}\beta$

$\text{th}(\alpha\pm\beta) = \frac{\text{th}\alpha \pm \text{th}\beta}{1 \pm \text{th}\alpha\,\text{th}\beta}$

$\text{sh}2\alpha = 2\text{sh}\alpha\,\text{ch}\alpha = \frac{2\,\text{th}\alpha}{1 - \text{th}^2\alpha}$

$\text{ch}2\alpha = \text{ch}^2\alpha + \text{sh}^2\alpha = 2\text{sh}^2\alpha + 1$

$= 2\text{ch}^2\alpha - 1 = \frac{1 + \text{th}^2\alpha}{1 - \text{th}^2\alpha}$

$\text{th}2\alpha = \frac{2\,\text{th}\alpha}{1 + \text{th}^2\alpha}\ ;\ \text{cth}2\alpha = \frac{1 + \text{cth}^2\alpha}{2\,\text{cth}\alpha}$

$\text{sh}\alpha \pm \text{sh}\beta = 2\,\text{sh}\left(\frac{\alpha\pm\beta}{2}\right)\text{ch}\left(\frac{\alpha\mp\beta}{2}\right)$

$\text{ch}\alpha + \text{ch}\beta = 2\,\text{ch}\left(\frac{\alpha+\beta}{2}\right)\text{ch}\left(\frac{\alpha-\beta}{2}\right)$

$\text{ch}\alpha - \text{ch}\beta = 2\,\text{sh}\left(\frac{\alpha+\beta}{2}\right)\text{sh}\left(\frac{\alpha-\beta}{2}\right)$

$\text{th}\alpha \pm \text{th}\beta = \frac{\text{sh}(\alpha\pm\beta)}{\text{ch}\alpha\,\text{ch}\beta}$

$(\text{ch}\alpha \pm \text{sh}\alpha)^n = \text{ch}n\alpha \pm \text{sh}n\alpha$